it is reasonable to expect that multidimensional systems will require more complex procedures for choosing the fading factor if for no other reason than the problems of observability and controllability that are attendant to such systems." We agree that choosing the fading factor adaptively is likely to prove a very complex problem. This seems to contradict the effectiveness and simplicity requirement of the filter.

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Comment on "Choked Flow: A Generalization of the Concept and Some Experimental Data"

Anthony M. Agnone*
New York University Aerospace Laboratory, Bronx, N.Y.

In Ref. 1 there is derived the chocking condition for two or more streams flowing through a De Laval type duct. This condition is arrived at from the physical requirement that for a given total mass flow rate the total equivalent momentum flux is a minimum; or alternatively, for a given total equivalent momentum flux the total mass flow rate is a maximum.

The same condition is derived here without having to resort to physical requirements, that is, on a purely mathematical basis. The choking condition is derived from the requirements at the singular points of the conservation equations. The assumptions made here are the same as Ref. 1, specifically, that the flow is steady, one dimensional and the gases are assumed ideal.

Following the development of Ref. 2, the conservation equations may be written as follows. The Mach number variation of each stream is given by

$$dM_i = -\frac{\{1 + [(\gamma_i - 1)/2]M_i^2\}}{1 - M_i^2}\frac{M_i}{A_i}dA_i + \sum_{j=1}^{N}\frac{f_j(\gamma_i,M_i)}{1 - M_i^2}\frac{M_i}{2}\frac{dZ_j}{Z_j} \qquad i = 1,2,\ldots,n$$

where the functions $f_j(\gamma_i,M_i)$ are listed in Table 8.1 of Ref. 2 and the dZ_j represent the forcing terms such as heat addition, mass injection, friction etc.... Since the pressure of all the streams is the same, i.e., $p_i = p$, the area variation of each stream may be calculated from

$$dA_i = \frac{1 - M_i^2}{\gamma_i M_i^2}\frac{A_i}{p}dp - \frac{A_i}{\gamma_i M_i^2}\sum_{j=1}^{N} g_j(\gamma_i,M_i) \times \frac{dZ_j}{Z_j} \qquad i = 1,2,\ldots,n$$

where the $g_j(\gamma_i,M_i)$ are also listed in Table 8.1 of Ref. 2. The area constraint relation

$$\sum_{i=1}^{n} A_i = A \text{ or } \sum_{i=1}^{n} dA_i = dA$$

supplies the final equation of a system of $2n + 1$ equations for the $2n + 1$ unknowns dM_i, dA_i, dp.

The singular point of this system occurs when the determinant of the homogeneous system vanishes. It is evident from the preceding that the determinant is equal to

$$D = \begin{bmatrix} \frac{\gamma M^2}{1 - M_i^2}\frac{p}{A_i} & & 0 & 1 \\ & \ddots & & \vdots \\ 0 & & \frac{\gamma_n M_n^2}{1 - M_n^2}\frac{p}{A_n} & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix}$$

After expansion by the nth column and then the resulting cofactors by their respective r-th column, this can be shown to be equal to

$$D = \sum_{i=1}^{n}\left(\prod_{\substack{i=1 \\ i\neq r}}^{n}\frac{\gamma_i M_i^2}{1 - M_i^2}\frac{p}{A_i}\right) = \left(\prod_{i=1}^{n}\frac{\gamma_i M_i^2}{1 - M_i^2}\frac{p}{A_i}\right) \times \left(\sum_{r=1}^{n}\frac{1 - M_r^2}{\gamma_r M_r^2}\frac{A_r}{p}\right)$$

The requirement that the determinant vanishes gives the choking condition as

$$\sum_{i=1}^{n}\frac{1 - M_i^2}{\gamma_i M_i^2}A_i = 0$$

This is seen to be the same result as obtained in Ref. 1. Given this condition and with the mass flux fixed, it can be shown that the momentum flux

$$J = \sum_{i=1}^{n}(pA_i + \dot{m}_i u_i)$$

is a minimum, i.e., $dJ = 0$ when $dZ_j = 0$.

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* Assistant Research Scientist. Associate Member AIAA.

Reply by Authors to A. M. Agnone

Harold J. Hoge* and Ronald A. Segars†
U. S. Army Natick Laboratories, Natick, Mass.

It is interesting to see that the conditions of generalized choking can be derived in different ways. We thank Mr. Agnone for his straight-forward mathematical treatment. After our paper appeared, Bernstein, Heiser, and Hevenor[1] also published a paper on generalized choking. In some respects their method of attack is intermediate between Mr. Agnone's and ours.

Reference

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* Head, Thermodynamics Group, Pioneering Research Laboratory.

† Physicist, Physical Chemistry Section, Food Laboratory.